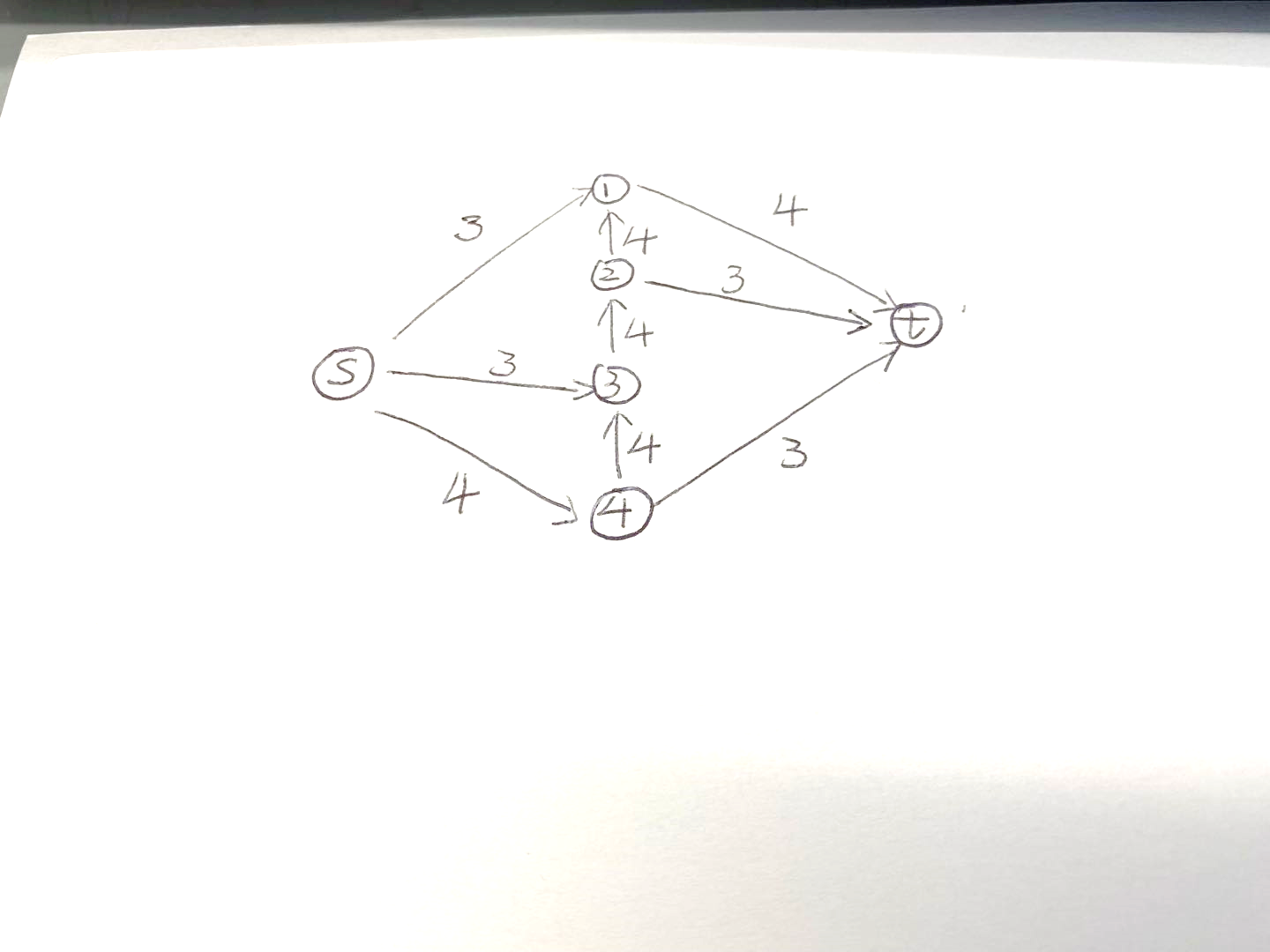
**CS5820 HW-6**

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1. Forward-Edge-Only Augmentations

The statement is false. Counter example is in below:



The maximum flow according to FF algorithm:

,

,

,

,

Total is 10

The forward-edge-only max-delta algorithm:

,

Total is

2. Finding minimum cut closest to t

(1) Algorithm

Build a residual graph based on the maximum value flow computing results.

Initialize set

While there exists an edge which and

(In other word, while there exists that is reachable to in )

Remove from

Add to

Endwhile

Return

(2) Running time

Build residual tree: the worst case contains 2m edges in residual graph, so it would take O(2m)

Initialize set , because totally n nodes

While loop: In each iteration, obviously Because there are at most 2m edges in residual graph, so iteration time is Hence while loop is totally

Totally, running time is , (because ).

(3) Correctness

1. is a minimal capacity cut.

Proof: Note v(f) as the maximum flow from s to t that has been calculated. According to the Lemma in the class, for any s-t cut , , where is the capacity of the s-t cut. Now we consider all edges connecting A and B, according to our algorithm, u must not be reachable to t in . If and , , because otherwise u would be reachable to t in . If and , . Hence, . Because for any s-t cut , , hence. is a minimal capacity cut.

2. In has as many nodes as possible.

Proof: Proving above claim is the same as prove has as few nodes as possible. Hence, we can prove this by contradiction. Assume there exist another minimum s-t cut and . Then there must exist a node that . According to the algorithm, must be reachable to in , so there must exist a path from to in . In this path, there must exist an edge . If , because otherwise won’t be in . Then . If , , because otherwise won’t be in . Then . In both cases, , hence can not be a minimal capacity cut. has the maximum and minimum .